Prediction of Inert Turbulent Swirl Flows

David G. Lilley*
Cranfield Institute of Technology, Cranfield, Bedford, England

Numerical finite-difference predictions are made of inert turbulent boundary-layer swirling flows. A variety of turbulence models are considered and a nonisotropic model is found to show more realistically the effects of swirl on jet development. Gross effects may be represented by an extended Prandtl mixing length model but constants appearing do not exhibit universality. This deficit is partially overcome by the use of an algebraically-modeled, nonisotropic energy-length turbulence model. The Richardson number and the local swirl number play important parts in linking the $r\theta$-shear with the rx-viscosity and the nonisotropy of the turbulent viscosity.

Nomenclature

A, B, C	
a, b, c, d	= coefficients in finite difference equations
a	= apparent origin distance
C	= constants in energy-length turbulence model
d	= nozzle diameter
E, I	= exterior and interior edges of boundary layer
G	= axial flux of momentum
K_e	= entrainment constant
K_1	= axial velocity decay constant
k	= kinetic energy of turbulence = $(\overline{u'^2}+\overline{v'^2}+\overline{w'^2})/2$
k_u	= axial velocity error curve constant
l	= Prandtl mixing length
L	= length scale of turbulence
m	= mass flow rate
$\dot{m}''$	= mass transfer rate across a boundary
p	= time-mean pressure
Ri	= Richardson number
r	= radial coordinate
S	= swirl number Eq. (14)
S_x	= local swirl number Eq. (13)
t	= time
u	= time-mean axial velocity
v	= time-mean radial velocity
w	= time-mean swirl velocity
x	= axial coordinate
δx	= forward step-length
θ	= polar coordinate
λ	= mixing length parameter
μ	= turbulent viscosity
ξ	= nondimensional radial coordinate = $r/(x+a)$
ρ	= time-mean density
σ	= Prandtl-Schmidt number
τ	= turbulent stress tensor
ϕ	= general dependent variable
ψ	= stream function
ω	= nondimensional stream function

Subscripts

o	= value at orifice of jet
rx, etc	= rx-component of second-order tensor, etc.
x	= relating to value at particular axial station
x, θ	= relating to directions x, θ
0.5, 0.05, 0.01	= position where u/u_m = 0.5, 0.05, 0.01, respectively

Superscripts

$(\,)'$	= turbulent fluctuating component
$(\bar{\,})$	= time-average

Presented as Paper 72-699 at AIAA 5th Fluid and Plasma Dynamics Conference, Boston, Mass., June 26–28, 1972; submitted July 18, 1972; revision received January 22, 1973. The author would sincerely like to thank the following for helpful discussions on swirl, prediction procedures and turbulence models: N. A. Chigier, M. L. Koosinlin, B. E. Launder, F. C. Lockwood, and W. Rodi.

Index categories: Boundary Layers and Convective Heat Transfer—Turbulent; Jets, Wakes and Viscid-Inviscid Flow Interactions.

* Senior Research Officer, School of Mechanical Engineering. Member AIAA.

I. Introduction

THE computational solution of turbulent swirling flows is of considerable interest to aerodynamicists. To designers of combustion chambers and industrial furnaces swirl is generated to assist in the stabilization of the flame and to promote rapid mixing. To aircraft designers unwanted swirl presents a hindrance by causing trailing vortices which occur when the separated flow over the wings rolls up into vortex cores at the wing tips. Experimental studies[1–6] show that swirl has large-scale effects on turbulent flows; jet growth, entrainment and decay being affected by the degree of swirl imparted to the flow. The swirl strength determines the degree of upstream influence. A strongly swirling jet flow (approximately $S \geqq 0.6$) possesses strong radial and axial pressure gradients in the region of the orifice; the axial one is sufficient to cause the interesting and useful effect of a recirculation zone to be set up. Weaker swirl does not cause such strong curvature of the streamlines as to result in recirculation and a boundary-layer jet flow results.[1,3–5] This paper is concerned with a direct finite-difference solution to inert turbulent free swirling jet flows of the latter type in stagnant surroundings.

Recent considerable progress in numerical solution procedures[7–10] has ensured that the governing equation system may be solved fairly readily. The main problem now is the simulation of the flow—the requirement of a model for the turbulent stresses. The turbulent stresses can be specified in terms of mean quantities already in the system or in terms of further unknowns with correspondingly further equations. Specification can be directly or indirectly by way of the exchange coefficients.[11,12] Isotropy of exchange coefficients has generally been assumed. Up to now constants in current energy-length models have not been determined for swirling flows or extended to portray nonisotropy; nor would their use yet be justified because of the boundary-layer type of the flow studied here until simple extensions of theories of the mixing length type are shown to be inadequate. Since recent works[5,6,15–16] dispute isotropy assumptions for swirling flows, this paper employs initially a nonisotropic extension of Prandtl's mixing length model and subsequently explores the possibility of a nonisotropic energy-length model.

This paper† describes some of the recent advances in the prediction of boundary-layer swirl flows with various turbulence models and allowance for the nonisotropy. A stable, accurate and economical numerical finite-difference prediction procedure

† Further details are in the original text.[28]

is used.[17,18] Solutions are generated for the free turbulent swirling jet in stagnant surroundings and compared with experimental data. Appropriate values of parameters occurring in the turbulence models used are given, so that flows of this type may now be satisfactorily predicted.

II. Basic Equations

The basic turbulent stress (Reynolds) equations of conservation of mass and momentum are assumed to hold for the time-mean variables with only the turbulent contributions to the stresses (the molecular contributions being negligibly small in fully turbulent free flow). They are written in a cylindrical polar coordinate system (x, r, θ) and the motion is assumed to be quasi-steady and axisymmetric with no external force. Invoking boundary-layer assumptions they become[18]

$$(\partial/\partial x)(\rho u)+(1/r)(\partial/\partial r)(r\rho v) = 0 \quad (1)$$

$$\rho(u\,\partial u/\partial x+v\,\partial u/\partial r) = (1/r)(\partial/\partial r)(r\tau_{rx})-\partial p/\partial x \quad (2)$$

$$\rho(u\,\partial w/\partial x+v\,\partial w/\partial r) = (1/r^2)(\partial/\partial r)(r^2\tau_{r\theta})-\rho vw/r \quad (3)$$

$$\rho w^2/r = \partial p/\partial r \quad (4)$$

It may be shown that the turbulent stresses are related to correlations of turbulent fluctuations, measured in hot-wire anemometry, by the relations

$$\tau_{rx} = -\rho\overline{u'v'} \quad (5)$$

$$\tau_{r\theta} = -\rho\overline{v'w'} \quad (6)$$

The equation system is not closed. Prior to solution for p, u, v and w (ρ is constant) constitutive assumptions must be made for the turbulent stresses. Suitable turbulence models to effect closure are discussed in Sec. III, many of them, by analogy with laminar flows, utilizing an extension of Newton's stress-strain relation with variable turbulent viscosity. If isotropy is not assumed the extension takes the form

$$\tau_{rx} = \mu_{rx}\,\partial u/\partial r \quad (7)$$

$$\tau_{r\theta} = \mu_{r\theta}\, r(\partial/\partial r)(w/r) \quad (8)$$

$\mu_{r\theta}$ is related to μ_{rx} via an $r\theta$-viscosity number defined (by analogy with Prandtl-Schmidt numbers) by

$$\sigma_{r\theta} = \mu_{rx}/\mu_{r\theta} \quad (9)$$

Gross transport integrals across the mixing layer lead to

$$G_x = \int_0^\infty (\rho u^2+p-p_\infty)r\,dr \quad (10)$$

$$G_\theta = \int_0^\infty \rho uwr^2\,dr \quad (11)$$

$$m = \int_0^\infty \rho ur\,dr \quad (12)$$

G_x is the constant axial flux of axial momentum, comprised of momentum and pressure terms; G_θ is the axial flux of angular momentum, constant for a swirling jet emerging into otherwise-undisturbed surroundings. The mass flow rate m of course increases with x as the jet entrains mass.

Since G_x and G_θ are invariants in a swirling jet with zero circulation they can be used to characterize the jet by defining a strength. A local nondimensional parameter called the local swirl number may be taken as

$$S_x = G_\theta/(G_x r_{0.01}) \quad (13)$$

and at any axial position it characterizes the effect of rotation on the flow. In a swirling jet it decreases with axial distance as the jet spreads from its initial orifice value of

$$S = G_\theta/(G_x d/2) \quad (14)$$

This jet constant S is called the swirl number of the jet. Note that strong swirl S_x always decays progressively with axial distance, the asymptotic case always being that of a nonswirling flow.

III. Turbulence Models

Closure of the time-mean equation system is effected by means of a turbulence model and models are generally classified according to the shear-stress hypothesis (whether or not turbulent viscosities are introduced) and the number of extra differential equations to be solved. Reviews of previous and current work are available with assessments.[11,12] If introduced, turbulent viscosities have generally been assumed isotropic until recently, even in flows with swirl, but recent experimental,[5,6] prediction[13,14,18] and inverse[16] works have disputed isotropy assumptions for swirling flows.

Mixing Length Extensions

Various extensions of Prandtl's mixing length theory to flows with swirl have been proposed, the task being to link the $r\theta$-shear with the rx-viscosity of the axial equation and to allow for the nonisotropy of the viscosity.[14,16–21] Here the former task is accomplished by taking the rx-viscosity proportional to the second invariant of the mean flow rate of deformation tensor and the latter by use of a variable $r\theta$-viscosity number. Thus

$$\mu_{rx} = \rho l^2\{(\partial u/\partial r)^2+[r(\partial/\partial r)(w/r)]^2\}^{1/2} \quad (15)$$

$$\mu_{r\theta} = \mu_{rx}/\sigma_{r\theta}, \quad l = \lambda r_{0.05}, \quad \lambda = 0.08\,(1+\lambda_s S_x)$$

where $\sigma_{r\theta}$ and λ_s are constants or functions of S_x.

The factor $(1+\lambda_s S_x)$ accounts for the change in the length scale due to swirl and is analogous to the Monin-Oboukhov formula

$$l = l_o(1-\beta Ri) \quad (16)$$

which has been suggested[19–21] as a simple approximate means of correlating the effect of streamline curvature and centripetal accelerations on the mixing length. β is an adjustable parameter and Ri the Richardson number which can be regarded as

$$Ri = \frac{(2w/r^2)\,\partial/\partial r(rw)}{(\partial u/\partial r)^2+[r(\partial/\partial r)(w/r)]^2} \quad (17)$$

Energy-Length Models

More recent work on calculating turbulent flows has postulated that the turbulence may be adequately described by two quantities: the kinetic energy k and the length scale L (Refs. 11, 12, 13, and 22–24). The relation

$$\mu_{rx} = \rho k^{1/2}L \quad (18)$$

is used and two extra differential equations are required, one for k itself and the other for any variable $z = k^m L^n$ These models exhibit greater universality than the mixing length model for flows without swirl, transport effects on turbulent viscosity can be accounted for and the length scale, being the outcome of a differential equation, does not have to be given an ad hoc distribution. The differential equations for k and z are developed by a combination of physical reasoning and intuitive guess work. In common with Rotta,[22] Rodi,[23] and Koosinlin[14] the z equation is taken for $z = kL$. Their work leads to fully-modeled equations for k and kL. These are taken as

$$\rho\left(u\frac{\partial k}{\partial x}+v\frac{\partial k}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\mu_{rx}}{\sigma_k}\frac{\partial k}{\partial r}\right)+\mu_{rx}\left(\frac{\partial u}{\partial r}\right)^2 + \mu_{r\theta}\left[r\frac{\partial}{\partial r}\left(\frac{w}{r}\right)\right]^2 - C_D\rho k^{1.5}/L \quad (19)$$

$$\rho\left(u\frac{\partial kL}{\partial x}+v\frac{\partial kL}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\mu_{rx}}{\sigma_k L}\frac{\partial kL}{\partial r}\right)+C_B L\left\{\mu_{rx}\left(\frac{\partial u}{\partial r}\right)^2 + \mu_{r\theta}\left[r\frac{\partial}{\partial r}\left(\frac{w}{r}\right)\right]^2\right\} - C_S\rho k^{1.5}+C_R\rho Ri k^{1.5} \quad (20)$$

where Ri is the Richardson number as previously defined. The body force, the last term in the kL-equation, characterizes the effect of rotation on the turbulence structure, extra kL being generated and hence higher viscosity. Values of the constants for nonswirling round jet flows are given by Rodi[23] as $C_D = 0.055$, $C_B = 0.98$, $C_S = 0.0397$, $\sigma_k = 1.0$, $\sigma_k L = 1.0$. These are retained and the parameter C_R chosen to best fit the experimental data. Nonisotropy is allowed for by retaining $\sigma_{r\theta}$ as a function of local swirl number S_x.

Observe that neglect of convection and diffusion in the k-equation and elimination of k with Eq. (18) yields Eq. (15) except for a factor $l/\sigma_{r\theta}$ multiplying the second term in the brackets where $l = L/C_D^{1/4}$. This gives weight to the extended form of Prandtl's local equilibrium model given in Eq. (15).

Stress Modeling

The preceding models are all of the turbulent viscosity type, and interest centers on the specification of μ_{rx}. More advanced alternatives never introduce μ_{rx} and specification of the stresses is direct from solution of the stress transport equations. Rodi conveniently tabulates the equations.[24] The shear stresses $\overline{u'v'}$ and $\overline{v'w'}$ required for the flow here are arguments of differential equations which contain other second and third order correlations. Rather than solve differential equations for higher-order correlations, modeling of the correlations in the two shear stress equations is often preferred, this modeling involving u, w, k, and L. Hanjalic[25] has developed a three-equation turbulence model for $\overline{u'v'}$, k and $k^{1.5}/L$ suitable for nonswirling flows. Koosinlin[26] has presented partially modeled equations for the two shear stresses needed for boundary-layer swirl flows. The problem of optimizing the values of constants appearing is considerable and the model is of little immediate use.

Algebraic Stress Modeling

On the assumption of local equilibrium, convection and diffusion of shear stress are neglected and the shear stress differential equations reduce to algebraic equations.[26,27] Koosinlin[26] obtains, after introducing the notion of turbulent viscosity [Eqs. (5–8)] for convenience

$$\mu_{rx} = \rho(C/C_D)\cdot L\cdot(\overline{v'^2}/k^{1/2}) \tag{21}$$

$$\mu_{r\theta} = \mu_{rx}(1-\beta Ri) \tag{22}$$

where

$$\beta = [(2-C_2)/(1-C_2)](\overline{w'^2}/\overline{v'^2}-1)$$

$$Ri = (w/r)/[r(\partial/\partial r)(w/r)] \quad \text{(a Richardson number)}$$

If the assumption $\overline{v'^2}\alpha k$ is made $\mu_{rx}\alpha\rho k^{1/2}L$ so that the theory provides a nonisotropic extension of the usual energy-length model. The function $(1-\beta Ri)$, a natural outcome of the analysis, is the Monin-Oboukhov proposal. Experimental data for the ratio $\overline{w'^2}/\overline{v'^2}$ vary with swirl number, axial and radial position. Pratte[5] reports values of order unity for $S = 0.3$ whereas Allan[6] reports values between 2 and 10 for $S = 0.6$.

This analysis of the modeled equations for the shear stresses in a swirling boundary layer reveals directly the nonisotropic nature of turbulent viscosity. The resulting algebraic equations for $\overline{u'v'}$ and $\overline{v'w'}$ combined with differential equations for k, kL, $\overline{v'^2}$, and $\overline{w'^2}$ form a four-equation turbulence model which has yet to be developed.

As a first stage, and analogous to the method used with the mixing length concept, the nonisotropy will be allowed for through the use of the local swirl number S_x. The k-kL model is used to obtain the primary viscosity component. Thus

$$\mu_{rx} = \rho k^{1/2}L \quad \text{and} \quad \sigma_{r\theta} = \text{function of } S_x \tag{23}$$

where the function of S_x is to be deduced empirically.

IV. The Prediction Procedure

The governing partial differential equations are quasilinear, parabolic and strongly coupled. A simple quick forward-marching numerical prediction procedure may be used and, accordingly, the equations are solved by use of the fully implicit Patankar-Spalding finite-difference procedure. The technique is stable, accurate and economical and is well-documented elsewhere.[10] Modifications of the basic procedure to solve the swirl equation satisfactorily have been given[10,17,18] together with program listings. Only highlights of the technique need be given here. A nondimensional stream function

$$\omega = (\psi-\psi_I)/(\psi_E-\psi_I) \tag{24}$$

where

$$\psi(r) = \int_0^r \rho ur\,dr \text{ (for fixed } x)$$

is employed as the independent variable across the layer. The main novelty lies in the choice of grid, which adjusts its width automatically at each forward step so as to conform to the thickness of the layer in which significant variations are present. In this von Mises coordinate system the equations possess the common form

$$\partial\phi/\partial x+(a+b\omega)\partial\phi/\partial\omega = (\partial/\partial\omega)(c\,\partial\phi/\partial\omega)+d \tag{25}$$

$$(\phi = u,\ rw,\ k,\ kL)$$

where the coefficients a, b, c, d have the usual significance.[10] Rather than solve the swirl equation in this form, the diffusion term of that equation is rewritten in terms of w/r instead of rw so as to eliminate the otherwise-troublesome source term d of that equation. The swirl equation becomes

$$(\partial/\partial x)(rw)+(a+b\omega)(\partial/\partial\omega)(rw) = (\partial/\partial\omega)[cr^2(\partial/\partial\omega)(w/r)] \tag{26}$$

A fully-implicit micro-integral method is used to obtain the corresponding finite-difference equation

$$\phi_i = A\phi_{i+1}+B\phi_{i-1}+C \tag{27}$$

in which A, B, and C are functions of the ω-differences, the values of the ϕ's at the upstream edge and the coefficients a, b, c, and d. These equations are then solved for each forward step by use of a simple successive-substitution technique and the solution is marched downstream as far as required, using initial data in accord with experimental findings.[4–6,23]

V. Predictions

This section presents the application of the prediction procedure to the inert turbulent free swirling jet in stagnant surroundings. Results generated with two extended turbulence models as described in Sec. III are compared with experimental data so as to establish values of additional constants and functions appearing in the models. Velocity decays and the fitting of parameters are considered first for each turublence model separately, then predictions of over-all effects of swirl on the flow are considered together.

The Mixing Length Model

The extended Prandtl mixing length model Eq. (15) with parameters λ_s and $\sigma_{r\theta}$ has been used to generate solutions. By individually varying each of λ_s and $\sigma_{r\theta}$, the effect of each individually on the flow may be deduced. Longitudinal variations with λ_s are shown for the case $S = 0.4$ and $\sigma_{r\theta} = 1.0$ in Fig. 1. It is seen that there is a progressive downward trend of both u_m and w_m as λ_s increases and that a value of near 0.6 gives a good fit against experiment. A similar result is obtained for other values of S and $\sigma_{r\theta}$. Alteration of $\sigma_{r\theta}$ from its unity value to other

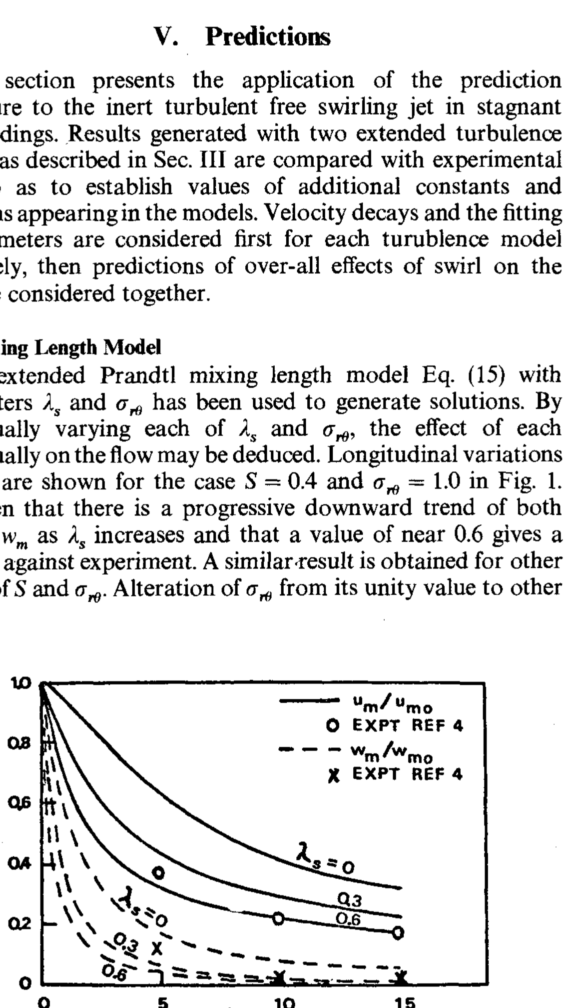

Fig. 1 Longitudinal decays for $S = 0.4$ and $\sigma_{r\theta} = 1.0$, mixing length model.

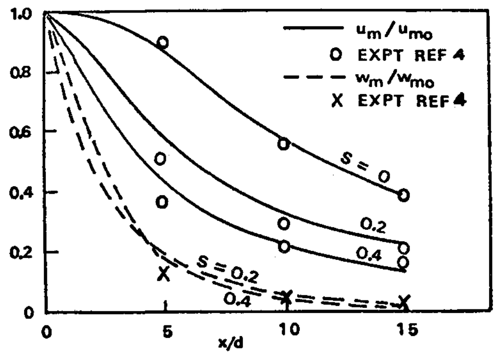

Fig. 2 Longitudinal decays with $\lambda_s = 0.6$ and $\sigma_{r\theta} = 1+5\ S_x^{1/3}$, mixing length model.

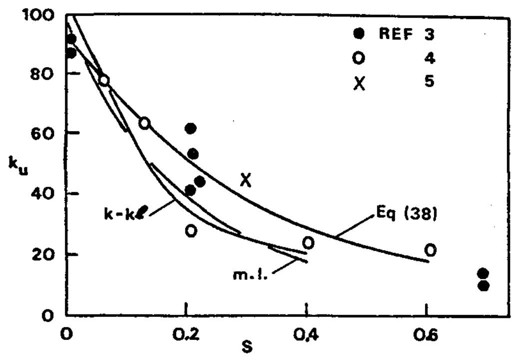

Fig. 4 Variation of axial velocity error curve constant with swirl.

constant values has little effect on u_m but a large effect on w_m: the larger its value the less rapid the decay of w_m and vice versa.

Consideration of these initial results enables a characterization of swirl to be deduced. Since $\sigma_{r\theta}$ has little effect on u, the more rapid decay of u_m with swirl can be obtained only through increased turbulence and a higher turbulent viscosity. The approximate characterization for the range $S = 0$ to 0.4 may be taken as

$$\lambda_s = 0.6 \tag{28}$$

To fit the w_m decay at $x/d = 10$ to experimental data with this value of λ_s, a constant value of $\sigma_{r\theta}$ may be taken in terms of the initial swirl number as

$$\sigma_{r\theta} = 1+5S \tag{29}$$

Predictions with these constant values give w_m decays slightly too much in the initial region but insufficient later. Better w_m decay with negligible effect on u_m is obtained with the $\sigma_{r\theta}$ large initially but tending to unity downstream. Further computer optimization yields the recommended variation of $\sigma_{r\theta}$, in terms of the local swirl number, as

$$\sigma_{r\theta} = 1+5S_x^{1/3} \tag{30}$$

Longitudinal variations for different degrees of swirl with this characterization are shown in Fig. 2. The decays of u_m and w_m compare well with experimental results. The progressive increase in the u_m decay as S increases and the approximately nonswirl-dependent w_m decay are clearly evident.

The Energy-Length Model

The predictions were repeated using the k-kL turbulence model as dictated by Eqs. (19, 20, and 23) with Rodi's round jet recommended constants for C_D, C_B, C_S, σ_k, and $\sigma_k L$ as given in Sec. III. Parameters or functions C_R and $\sigma_{r\theta}$ supposedly allow the effect of swirl on the turbulent viscosity and the nonisotropy to be catered for. In the first instance they were made simply

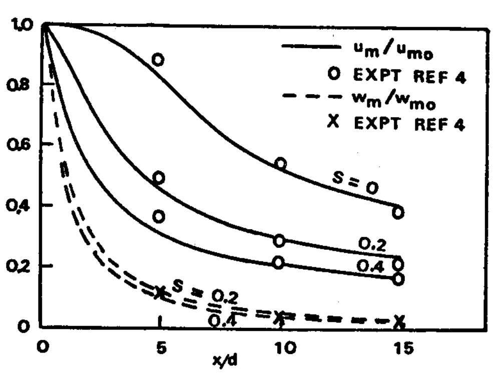

Fig. 3 Longitudinal decays with $C_R = 0.06$ and $\sigma_{r\theta} = 1+2\ S_x^{1/3}$, energy-length model.

constants and computations made with various combinations of values. The longitudinal effect of increasing C_R is to produce a progressive downward trend of both u_m and w_m. A value of C_R between 0.04 and 0.08 seems to predict the u_m decay fairly well with $\sigma_{r\theta} = 1$.

Increasing $\sigma_{r\theta}$ has some effect on the u_m decays but its greatest effect is to provide less rapid decay of w_m. This opposes the observation made with the mixing length model where the effect of $\sigma_{r\theta}$ is largely restricted to w_m. It is a consequence of the stronger coupling of w and $\mu_{r\theta}$ in the turbulence model.

Computer optimization to fit the experimental data at $x/d = 10$ yields the following recommendations for constant values of the parameters:

$$C_R = 0.06 \tag{31}$$

$$\sigma_{r\theta} = 1+3S \tag{32}$$

Similar u_m decays but with better predictions of w_m were obtained with $\sigma_{r\theta}$ a function of local swirl number, $\sigma_{r\theta}$ being larger initially but tending to unity downstream. The recommended function, similar to that for the mixing length model, is

$$\sigma_{r\theta} = 1+2S_x^{1/3} \tag{33}$$

and longitudinal results with this are shown in Fig. 3. Again, the decays shown compare well with experimental results.

Gross Effects

The results show that the effect of swirl on jet decay may be adequately predicted with two turbulence models and recommendations have been given for constants and functions appearing in them. The primary use of swirl in a jet is to increase the angle of spread and the rate of decay of axial velocity; one is not so much concerned with the swirl velocity field as one is concerned with the effect of the initial degree of swirl on the subsequent flow. For example the jet with swirl number 0.4 is almost twice as wide as its nonswirling counterpart. This is of considerable interest in engineering applications and, for practical purposes, it is necessary to know the variation with swirl of constants concerned with jet development. Experimentalists have evaluated four such constants concerned with the axial velocity error curve (k_u) (for the axial velocity profile), axial velocity decay (K_1), entrainment (K_e) and half-angle (α) (Refs. 1–5). These are defined by

$$u/u_m = \exp(-k_u \xi^2) \tag{34}$$

$$u_m/u_{mo} = K_1\, d/(x+a) \tag{35}$$

$$m/m_o = K_e\, x/d \tag{36}$$

$$\tan\alpha = r_{0.5}/(x+a) \tag{37}$$

Predicted profiles are now considered. The u profile changes from the near plug flow initial one to a gaussian one of the fully developed region some few diameters downstream. The w profile changes from a near solid body rotation to a Rankine free-forced vortex type. In order to compare the predicted effects of swirl on the flow, the aforementioned four constants were evaluated from both sets of predictions in the fully developed

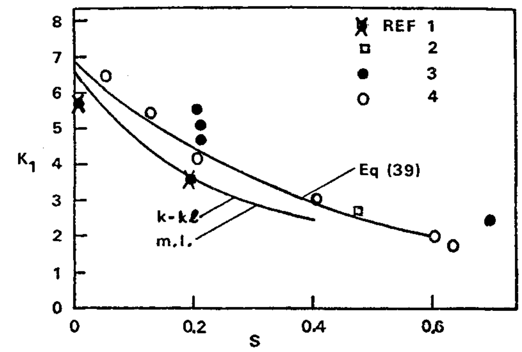

Fig. 5 Variation of axial velocity decay constant with swirl.

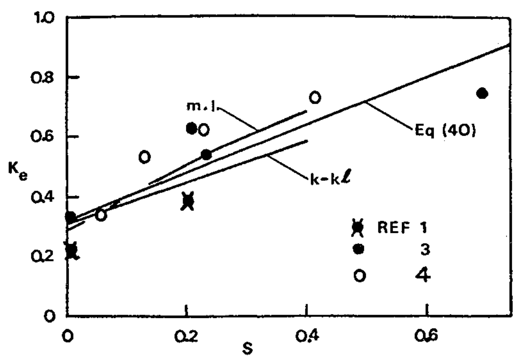

Fig. 6 Variation of entrainment constant with swirl.

region for the three degrees of swirl $S = 0.0$, 0.2 and 0.4. Figures 4 to 7 show and compare the predictions with the experimental evidence; also shown are the recommended variation curves of Chigier and Chervinsky[4] for weak and moderate swirl

$$k_u = 92/(1+6S) \tag{38}$$

$$K_1 = 6.8/(1+6.8S^2) \tag{39}$$

$$K_e = 0.32+0.8S \tag{40}$$

$$\alpha = 4.8+14S \tag{41}$$

Results agree well with the data.

These figures show that the effect of swirl on jet growth, entrainment and decay may be predicted well by either of the two turbulence models. The discrepancy seen in Fig. 5 between the predicted K_1's and Eq. (39) is because of a slightly different definition of K_1 used by Chigier.

VI. Discussion

Mixing Length and Energy-Length Models

Both models predict the data quite adequately and, from the practical viewpoint, there would appear to be no reason to discard the simpler mixing length model for the flow considered here. This must be considered purely fortuitous since the flow is nearly an equilibrium one and the defects of the mixing length model are most apparent in flows with strong historical effects. The greater universality of the energy-length model is its main commendation: less nonuniversal parameters need specifying prior to its use. It represents a further step towards the goal of complete prediction.

Unfortunately both models in their original form require modification with further constants and parameters in order to make satisfactory predictions of the effect of swirl on axial velocity decay, that is the linkage of the $r\theta$-shear with the rx-viscosity. These modifications, together with appropriate constants and functions have been deduced in Sec. V and are represented by Eqs. (28–33).

Nonisotropy

Another defect of both models is the crude representation of the nonisotropy of the turbulence and the predictions of swirl velocity decay. The nonisotropy does not affect greatly the axial velocity decay. Indeed Siddhartha[17] reports excellent predictions starting from the axial station $x/d = 2.0$ with an isotropic model. Results presented in Sec. V show that the nonisotropy is especially intense in the initial region and constants or simple functions of the local swirl number were all that were allowed for so that a constant value of $\sigma_{r\theta}$ was used at any particular axial station. Better results could no doubt be achieved in profile shapes if it was allowed to vary across the layer, being a function perhaps of the Richardson number as in Eq. (22). However it is felt that this is of minor importance as compared to the effect of the $r\theta$-shear on the rx-viscosity and that such sophistication would not be warranted until the fuii four equation algebraic stress model is used.

The nonisotropy, generated by Eqs. (29, 30, 32, and 33), with both models was found in general to confirm previous findings[16] that nonisotropy increases with the degree of swirl and is more marked in the initial region. The $\sigma_{r\theta}$ values were greater than unity whereas Koosinlin and Lockwood[14] (who predict nonswirling flow encountering rotating cylinders, cones and disks) find values less than unity. This is as expected for it is well known that a swirling flow in stagnant surroundings exhibits properties opposite to a nonswirling flow emerging into a vortex. The fact that the two turbulence models here required slightly different specifications of the nonisotropy is an admission of the limitations of attempts to predict these flows by current models.

Algebraic Modeling of Stresses

The nonisotropy introduced via the local swirl number and taken as constant across the layer must be regarded as but an intermediate step. A more sophisticated approach is necessary in a model possessing universality. Such models, introduced in Sec. III, are concerned with the direct calculation of the stress terms required in the axial and swirl equations. Rather than solve the stress differential equations, simplification via algebraic modeling would appear to be more fruitful in the short term. For nonswirling flows, Hanjalic and Launder[25] have had considerable success and it would seem that for flows like the one considered here, of the boundary-layer type with a marked degree of nonisotropy, progress is imminent.[26]

VII. Conclusions

A numerical finite-difference solution procedure now exists for boundary-layer turbulent flows. Solutions are generated with two turbulence models—one an extended Prandtl mixing length model and the other analogous to an algebraically-modeled energy-length model. Good predictions of the effects of swirl on

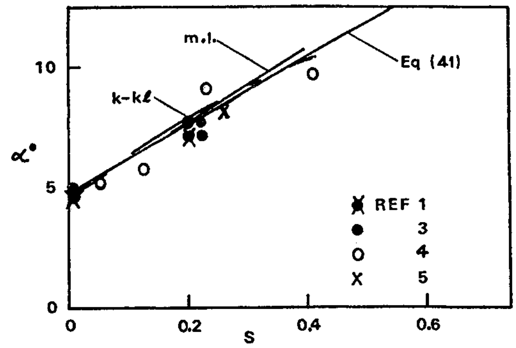

Fig. 7 Variation of jet half-angle ($u/u_m = 0.5$) with swirl.

jet growth, entrainment and decay may be made with either model after suitable modifications. In order to predict the axial velocity field, both models require a strong link between the $r\theta$-shear and the rx-viscosity. The swirl velocity field may be adequately predicted only if some nonisotropy of the turbulent viscosity is allowed for. The $r\theta$-viscosity number $\sigma_{r\theta}$ may be taken uniform across the flow, greater than unity and a function of local swirl number. The appropriate simulation is embodied in Eqs. (28–33) where the Richardson number and the local swirl number play important roles.

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